FOSS to Enhance Mathematics Education

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***Abstract*—In this paper, the necessity and some applications of FOSS in mathematical education are discussed. The importance and participation of technology in education is highlighted. Also some suggestions and examples for effective and efficient way of teaching mathematics with the support of technology through FOSS are explained.**

***Index Terms*—FOSS, Free Software, GeoGebra, LaTeX, Maxima, R, SAGE, Scilab.**

I. INTRODUCTION

INearly ages, usually the principle of education is developing basic inter-personal communication and literacy. Slowly, the purpose for pursuing education is varied with respect to the individuals or the group of people. After acquiring such basic abilities, they are improving the tendency to learn more complex skills and subjects like Astronomy, Natural Science, Mathematics etc.,. They were institutionalized the teaching and learning process because they felt the aim and knowledge of a group of people sustain from one generation to the next. Now a day various types of system of education occur all over the world. In most of the countries, it is compulsory to receive primary education. Based on this methodical education the world obtains good technology to grown-up. The augmentation of every nation based on the advancement of the technology. Also technology is an increasingly influential factor in education. Technology offers powerful teaching and learning tools like Multimedia, Virtual learning environment, Virtual Manipulative etc.,. Such tools demand new way, skill and understanding of teachers and students. The use of technologies is capturing the concentration of students in the classroom and in the assessment of students. The education takes on many forms, ranging forms, ranging forms formal chalk and talk based learning to self-directed learning and e-learning.

In mathematical education, even a good teacher as well as student takes much time to understand the theoretical content of a text book. They could not visualize the concept in their mind. Even it happens, it take two much of time. Also they are not good enough to explain the concept to others in class room environment. Also the delivering and understanding thecontent differs with respect to the teacher's and learner's knowledge. The environment also makes impact in the teaching and learning process. The participation of technology is essential to enhance mathematical education. Especially free and open source software (FOSS) is needed.

In this paper, we discuss that why does mathematical education need the support of technology? Any why does an application of FOSS essential to enhance the methodology of mathematical education? Some of the applications of FOSS are also discussed.

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II. WHY FOSS

The system of teaching and learning process is different in different regions all over the world. Now a days the designing a system of teaching and learning process is too complicated for all educational institutions. An effectual system may not produce good products. The technology is applied in the teaching and learning process. It is essential to use FOSS in the technology to optimize the time, money and level of understanding. Because of the teaching and learning process differs with respect to the teacher, learner and the environment. FOSS is over-come the difficulties of the immigration of traditional education environment to technology based education. Most of them are free of cost and easy to access the source code. The main thing is that it can be modified to increase the satisfaction of the teachers who use this. Also it or its modified version can be shared at free of cost for the benefit of teacher and learner.

It is essential that teachers should learn something about technology that adopted in their related subject. Because teaching three or four dimensions in mathematics, explain the inner parts of human body in zoology, motion of a planet in physics, financial market in administration, E-commerce in commerce etc., are not possible with the existing chalk and talk method. Effective teaching and efficient learning of these types of contents are possible only by using technology with in the class room. It is not mean that there is no need of teachers to teach this kind of subjects. It means that teacher should use technology with the class room of school and college to teach that.

"I hear and I forget; I see and I remember; I do and I understand" – Chinese proverb. Literally, Not hearing is not as good as hearing, hearing is not as good as seeing, seeing is not as good as mentally knowing, mentally knowing is not as good as acting; true learning continues up to the point that action come forth. FOSS is needed for the student and teacher to do many things to understand. It allows freedom do anything for education.

III. MATHEMATICAL FOSS

Especially the abstractness of mathematics like integration of a function, Riemann sum, plotting more than two dimension in analytical geometry, stereographic projection of complex plan, topological structures etc. The graphical interpretation is important to understand the abstractness of mathematics. Several FOSS can be used for research and academic related activities.

Some of them are listed below:

A. LaTeX –Document Preparing Software

B. GeoGebra–Interactive Geometry, Algebra and Calculus

C. Sage – Computer Algebra System

D. Scilab –Numerical Computing

E. R–Statistical Analysis

F. GNU Plot, Maxima, NetworkX, eXe– Others

IV. Computer Algebra System

The CAS technology is used in classes with the evolution of software and hardware program is too easy. However, the proportion of teachers, who use computers during their classes, is still quite low and it is hard to say that technology has started to be commonly used in classes. The representation which aim of the mathematics learning, contents and method, has various things. It is classified in five types: realistic, manipulative, illustrative, linguistic and symbolic[1]. When it changes a representation from a certain style into other styles, it is called translation. Translation in the mathematical studies is application of representation style in the teaching and learning of classroom mathematics. Not only the classroom lecture ,in the case of the problem solving, learners become easy to understand if learners can translate the expression methods properly. An advantage of CAS use is that an algebraic calculation is possible. The symbolic representation is an important aim of the mathematics learning.

A computer algebra system is a type of software package that is used in manipulation of mathematical formulae. The primary goal of a computer algebra system is to automate tedious and sometimes difficult algebraic manipulation tasks. The principal difference between a computer algebra system and a traditional calculator is the ability to deal with equations symbolically rather than numerically.

The first popular computer algebra system were Axiom,GAP, muMATH, Reduce and MAcysma; a popular copyleft version of Macsyma called Maxima is actively being maintained. As of today, the most popular commercial systems are Mathematica and Maaple, which are commonly used by research mathematicians, scientists and engineers. Freely available alternative include Sage which can act as a front-end to several other free and non-free CAS [1,3].

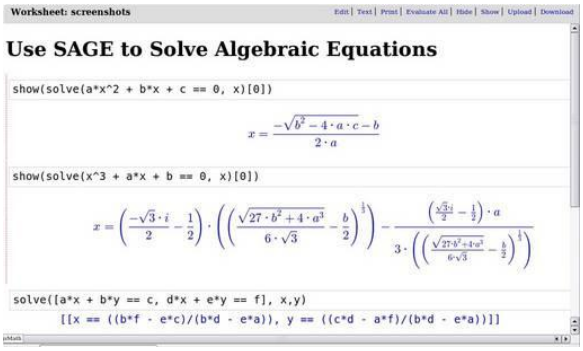

Fig. 1 The SAGE Note Book.

SAGE (System for Algebra and Geometry Experimentation) is mathematical software with features covering many aspects of mathematics, including algebra, combinatorics, numerical mathematics, number theory and calculus. It is sometimes called sage-math to distinguish it from other uses of the word. The originator and leader of the sage project, William Stein, is a mathematician at the University of Washington [4].

Sage uses the Python programming language, supporting procedural, functional and object-oriented constructs.Though sage is available in many ways for Linux operating system and users of Windows operating system currently have to use virtualization technology such as Virtual Box to run it. There are several mathematical packages contained in sage for Algebra, Algebraic Geometry, Arithmetic Geometry, Calculus, Combinatorics, Linear Algebra, Graph Theory, Group Theory, Numerical Computation, Number Theory and Statistics.

V. LaTeX

LaTeX is a document markup language and document preparation system for the TeX typesetting program. It is not only for preparing documents and presentations that contains lot of mathematics and for effective usage of the FOSS like eXe, Moodle etc that indirectly support to enhance mathematics education.

LaTeX was originally written in the early 1980s by Leslie Lamport [6]. It is to be created by using some form of text editor like Texniccenter, Winshell(closed source), Winedit etc.,. '.tex' is the extension of the LaTeX file. This system offers programmable desktop publishing features and extensive facilities for automating most aspects of typesetting and desktop publishing, including page layout, numbering and cross referencing, tables and figures, algorithms, and bibliographies. It is also used to create presentations in pdf format. It can be arbitrarily extended by using the underlying macro language to develop custom formats. Traditional TeX will output a dvi file, which is usually converted to a PostScript file. But recently it has three types of output file options dvi, pdf and ps which have their own advantages of features available in their formats. It is platform independent. It can be open from any text editor.

MikTeX is an up-to-date implementation of TeX and LaTeX programs for Microsoft operating system. It is very easy to install. A complete set of additions like programs, fonts, colors, packages, styles etc., is available to help you for typesetting the documents. MiKTeXs integrated package manager installs the missing components from the Internet, if the document required it. MiKTeX package repository is updated regularly. It is always up-to-date.

TeXnicCenter is an integrated documentation environment (IDE) for LaTeX. Its innovative user interface, its powerful features, its large community (more than millions of users all over the world) make it the first choice for LaTeX authors working on the Microsoft Windows platform. And the fact that it is free and open source software under General Public License (GPL) is an added advantage. It can be used to create and write new LaTeX document, building the output in dvi, ps and pdf formats, viewing the output in respective viewers MiKTeXs previewer, Ghost view and Adobe Reader, Print, Configuration and Customization.

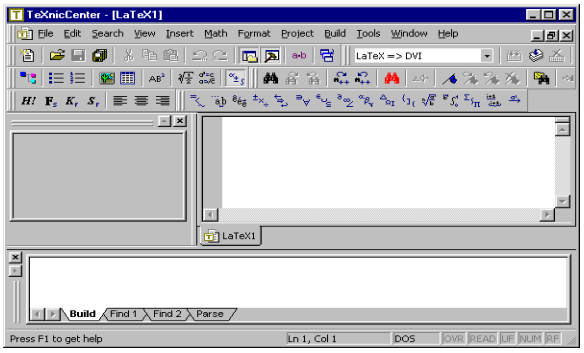

Fig. 2. TeXnicCenter.

There is a compulsory three lines in LaTeX document,

```
\documentclass{<class type>}
\begin{document}
\end{document}
```

The class types are article, book, thesis etc. The text, mathematical equations, image, picture, table etc whatever the content that typed only in between \begin{document} and \end{document}, will appear in the generated output file. The preambles like packages, user defined commands etc lies only in between \documentclass{<class type>} and \begin{document}.

VI. GeoGebra

GeoGebra [2, 5]. is one of the free mathematics software for learning and teaching mathematics. It is used to teach and learn not only mathematics, also used for some other subject. It is an interactive geometry, algebra and calculus application, intended for teachers and learners. Markus Hohenwarter [7] started this project in 2001.

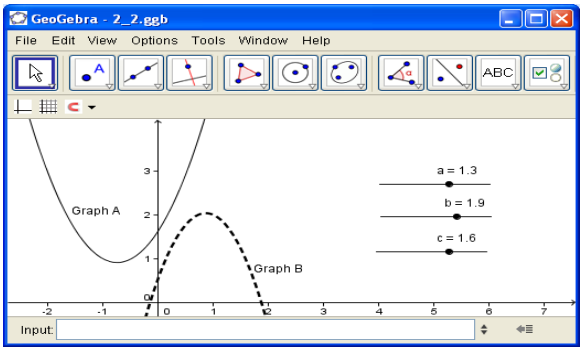

Fig. 3. Function and Graph in GeoGebra.

It can be used to make conjectures and prove geometric theorems. It allows us to create our own tools. It also used in class room environment to explain the definition, theorem, formulae etc. The constructed projects of GeoGebra can be exported in several formats like html, png, eps, GIF, PSTricks etc. Dynamic applets can be directly exported to a virtual learning environment such as Moodle. It also creates code that can be used in LaTeX files in order to create those same images that it generates. GeoGebra Tube is the official repository for the GeoGebra construction and GeoGebra related resources. The contents of this tube or modified contents of this tube can be shared. There is an option to export the constructed file directly from GeoGebra to its tube.

VII. Scilab

MATLAB is widely known standard numerical and scientific computation software because of its reliability, ease of use and support. Though, its main disadvantage is that it is a too costly alternative. In this way, there exist other free alternatives like Scilab and Octave, even though not as comfortable as MATLAB, provides some useful functionality and also offers two significant advantages: they are free and their source code is publicly available. Scilab can be an alternative for MATLAB.

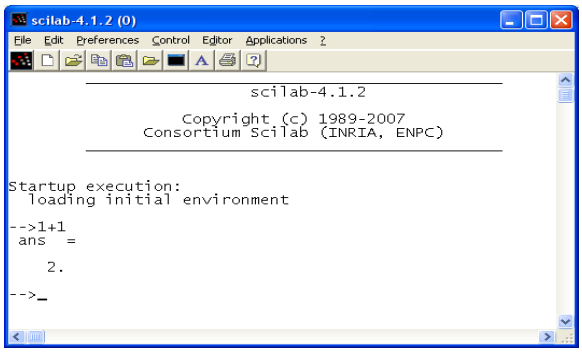

Fig. 4. Scilab.

Scilab is free open source software for numerical computation which includes hundreds of mathematical functions [8]. It has a high level programming language allowing access to advanced data structures, 2-D and 3-D graphical functions. It is widely used in high schools all over the world, in China, India or Japan for example. Some of schools, colleges and universities include Scilab in their curriculum. Also it can be used as a tool for teaching and learning mathematics. Students can work independently and responsibly while learning mathematical concepts. Freely available at school or at home, the use of Scilab by students and teachers is favored.

VIII. R

R is one of the free and open source software environments for statistical computing and graphics [9]. The R language is generally used among statisticians for developing statistical software and analyzing the data. It compiles and runs on a wide variety of UNIX platforms, Windows and Mac operating system. IT provides a wide variety of statistical such as clustering, classification, linear and non-linear modeling, classical statistical tests, time-series analysis etc., and graphical techniques, and is extremely graphical techniques, and is extremely extensible. It is an integrated suite of software facilities for data manipulation, calculation and graphical display.

Also, R is a well-developed, simple and effective programming language which includes conditionals, loops, user-defined recursive functions and input and output facilities. It has its own LaTeX-like documentation format, which is used to supply comprehensive documentation. The capabilities are extended through user-created packages, which allow particular statistical techniques, graphical devices, import / export capabilities, reporting tools etc. These packages are developed primarily in R and sometimes in JAVA, C and Fortan.

IX. EXAMPLES

To highlight the advantages and useful features, the experience of the other shared by the way of the following two examples. In the first example the abstract definition of integration is taken. The understanding level of learner and time consumption is optimized by using GeoGebra in teaching. In the second example is about LaTeX.

A. *Riemann Integration*

In mathematics, Riemann Integration

$$\int_l^u f(x)dx$$

is defined as follows,

A *partition* of an interval $[l,u]$ is a finite set of points $P=\{x_0,x_1,\dots,x_n\}$ such that $l=x_0<x_1<\cdots<x_n=u$. The partition P' is *finer* then the partition P if $P'\supseteq P$. A choice of T for the partition P is a finite set of points $t_1,t_2,\dots,t_n$ such that $x_{i-1}\le t_i\le x_i$ for each $1\le i\le n$. Then the partial sum

$$L(P,f)=\sum_{i=1}^{n}\inf f(t_i)\,(x_i-x_{i-1})$$

is defined as *Riemann Lower Sum*. And the partial sum

$$U(P,f)=\sum_{i=1}^{n}\sup f(t_i)\,(x_i-x_{i-1})$$

is defined as *Riemann Upper Sum*.

A bounded function defined as $f:[l,u]\longrightarrow\mathbb{R}$ and $\mathcal{P}=\mathcal{P}[l,u]$ is the set of all partition of $[l,u]$ then the upper and lower Riemann integrals are

$$\underline{\int_l^u} f(x)dx=\inf U(P,f) \quad \text{and} \quad \overline{\int_l^u} f(x)dx=\sup L(P,f)$$

respectively. The sup and inf taken over all the possible partitions. A bounded real valued function defined is said to be *Riemann-Integrable* on $[l,u]$, If

$$\underline{\int_l^u} f(x)dx=\overline{\int_l^u} f(x)dx.$$

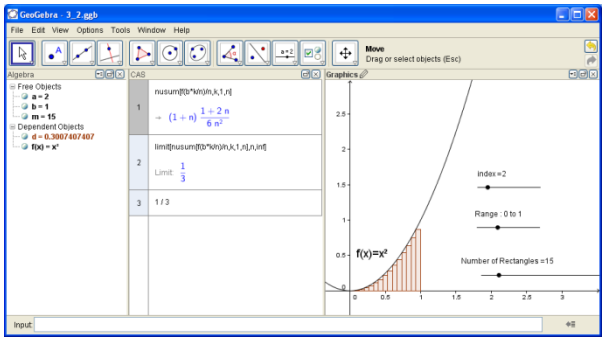

Fig. 5. Function and Riemann Integration.

The above definition is one of the best example which may not be teach with the better understanding of the students by using chalk and talk in the usual class room environment. For the effective and efficient teaching, the worksheet that created by using GeoGebra can be used. Refer the link http://www.geogebratube.org/student/m20636

The following table is the construction protocol [7] of the above .ggb file. It is easily obtained for any .ggb file by selecting View → Construction Protocol or by using the short cut key Ctrl+Shift+L.

Name	Definition	Value
Function f		f(x) = x³ - 5x² + 2x + 13
Line b		b: y = 0
Point A	Point on b	A = (1.44, 0)
Line c	Line through A perpendicular to b	c: x = 1.44
Point B	Intersection point of f, c	B = (1.44, 8.48)
Line d	Line through B perpendicular to c	d: y = 8.48
Point C	Intersection point of d, x = 0	C = (0, 8.48)
Text A_{}		$\mbox{A is the element }x \mbox{ of the domain of the function } f $
Text text2	"$f(x)=" + (FormulaText[f]) + "$"	$f(x)=x^{3} - 5 \; x^{2} + 2 \; x + 13$
Text C_{}		$ \mbox{C is the image of A } ,\mbox{ say } f(\rm A). $
Boolean Value e		e = false
Number u		u = 4
Number n		n = 20
Number l		l = -1.2
Text text1		Upper Limit
Text text3		Lower Limit
Text text4		Number of Partition
Number lrs	LowerSum[f, l, u, n]	lrs = 32.44
Number urs	UpperSum[f, l, u, n]	urs = 39.66
Number ri	Integral of f from l to u	ri = 36.09
Text text5	"=" + urs + ""	39.66
Text text6	"=" + ri + ""	36.09
Text text7	"=" + lrs + ""	32.44
Boolean Value a		a = false
Boolean Value g		g = false
Boolean Value h		h = false
Boolean Value i		i = false
Boolean Value j		j = false

Table. 1 Construction Protocol.

If the exercise is given to the student to change the value of n in this ggb file and report his or her observation

B. *LaTeX*

The same definition of Riemann integration is typed in the free and open source software LaTeX by using the following command lines.

```
\documentclass[10pt,twocolumn]{article}
\usepackage{amsmath}
\def\upint{\mathchoice%
    {\mkern13mu\overline{\vphantom{\intop}\mkern7mu}\mkern-20mu}%
    {\mkern7mu\overline{\vphantom{\intop}\mkern7mu}\mkern-14mu}%
    {\mkern7mu\overline{\vphantom{\intop}\mkern7mu}\mkern-14mu}%
    {\mkern7mu\overline{\vphantom{\intop}\mkern7mu}\mkern-14mu}% \int}
\def\lowint{\mkern3mu\underline{\vphantom{\intop}\mkern7mu}\mkern-10mu\int}
\begin{document}
In mathematics, Riemann Integration $\int\limits_l^u{f(x)dx}$ is defined as follows,\\
A partition of an interval $[l,u]$ is a finite set of points $P=\{x_0,x_1,\ldots,x_n\}$ such that $l=x_0<x_1<\ldots<x_n=u$. The partition $P^\prime \supset P$. A choice of $T$ for the partition $P$ is a finite set of points $t_1,t_2,\ldots,t_n$ such that $x_{i-1}\leq t_i\leq x_i$ for each $1\leq i\leq n$. Then the partial sum $L(P,f)=\sum\limits_{i=1}^n \inf f(t_i)(x_i-x_{i-1})$ is defined as Riemann Lower Sum. And the partial sum $U(P,f)=\sum\limits_{i=1}^n \sup f(t_i)(x_i-x_{i-1})$ is defined as Riemann Upper Sum.\\
A bounded function defined as $f:[l,u]\rightarrow R$ and $\mathcal{P}=\mathcal{P}[l,u]$ is the set of all partition of $[l,u]$ then the upper and lower Riemann integrals are $\upint_l^u f(x)dx=\inf U(P,f)$ and
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$\lowint_l^uf(x)dx=\sup L(P,f)$ respectively. The $\sup$ and $\inf$ taken over all the possible partitions. A bounded real valued function defined is said to be Riemann-integrable on $[l,u]$, If $\upint_l^u f(x)dx=\inf U(P,f)=\lowint_l^u f(x)dx=\sup L(P,f)$.

\end{document}
```

Fig. 6 shows the output of the above coding of LaTeX document. This is taken as an example for the explanation of the worksheet created by GeoGebra . But in this MS-Word document, it may not be typed as it is. Even though the author spent more time to get the support from online and offline resources, it was not possible to type. Then he compromised himself and started typing the definition that is in Example A.

In mathematics, Riemann Integration $\int_l^u f(x)dx$ is defined as follows,

A partition of an interval $[l,u]$ is a finite set of points $P = \{x_0, x_1, \ldots, x_n\}$ such that $l = x_0 < x_1 < \ldots < x_n = u$. The partition P' is finer then the partition P if $P' \supset P$. A choice of T for the partition P is a finite set of points $t_1, t_2, \ldots, t_n$ such that $x_{i-1} \leq t_i \leq x_i$ for each $1 \leq i \leq n$. Then the partial sum $L(P,f) = \sum_{i=1}^{n} \inf f(t_i)(x_i - x_{i-1})$ is defined as Riemann Lower Sum. And the partial sum $U(P,f) = \sum_{i=1}^{n} \sup f(t_i)(x_i - x_{i-1})$ is defined as Riemann Upper Sum.

A bounded function defined as $f : [l,u] \to R$ and $\mathcal{P} = \mathcal{P}[l,u]$ is the set of all partition of $[l,u]$ then the upper and lower Riemann integrals are $\overline{\int}_l^u f(x)dx = \inf U(P,f)$ and $\underline{\int}_l^u f(x)dx = \sup L(P,f)$ respectively. The sup and inf taken over all the possible partitions. A bounded real valued function defined is said to be Riemann-integrable on $[l,u]$, If $\overline{\int}_l^u f(x)dx = \inf U(P,f) = \underline{\int}_l^u f(x)dx = \sup L(P,f)$.

Fig. 6. Function and Riemann Integration.

The file size of the MS-Word (.doc) that contains the above same definition is 13.7KB but the file size of LaTeX (.tex) file is just 1.57KB. Suppose the large file is needed then there no alternative of LaTeX. Also it is easy to observe that the mathematical symbols integration, lower-integration, upper-integration and summation might not appear in the MS-Word by the comparison of these two file output. There are several online and offline resources available to learn and ask doubts about the commands of LaTeX. Most of the reputed journals asked the research papers in the tex format files. So, learning to prepare the document by using LaTeX is compulsory for the researchers. It is more comfortable for researchers to type their research paper and their PhD thesis in LaTeX.

Using LaTeX for document preparation and presentation is the only way to feel and understand the advantages of it. Not only for researchers but for a mathematics teacher it is necessary to know the LaTeX commands for the effective usage of many FOSS like eXe, Moodle, GeoGebra etc. LaTeX optimizes the output quality and style of the e-content, webpage, document, presentation etc., with lot of mathematical symbol and expressions in the better way.

X. Conclusion

Other than the above discussed Mathematical FOSS, there are plenty of free mathematical open source software like GNU Plot, Euler, FreeMat, NetworkX, Metamath, Numpy, Graph-tool etc.. Every FOSS has its unique features and common advantages that free of cost, allow accessing the source code, modifying and redistributing it. Our intention is to increase interactivity of students during their study in school or college. Computer simulations and computations have the advantage of allowing a student to make decision, and also to make errors. There is a possibility of learning in detail and depth by correcting the errors. The process of interactive teaching and learning through assessment, decision-making, and error correction creates a much stronger learning environment than passive instructions. Mathematical FOSS is the right choice to enhance the mathematical education such a way.

In other words, by educating mathematics in a way to make the students able to judge the use of the technology through FOSS depending on the situation, the worry on the insight will be avoided. These types of activities develop the students' basic calculation abilities and algebraic insight, solving numerical problems with hand calculations becomes most efficient. Moreover, through this kind of experience, students can evaluate mathematics from a broader outlook. And students can sense the power of their knowledge in mathematics and their relation with mathematics and technology. Also, mathematical FOSS makes the class room environment more comfortable for teachers. Due to it optimize the timing; there is no problem to complete the curriculum within given time with the higher satisfaction level of the students. These are all the reasons to recommendfree mathematical Open Source Software for the teachers and students.

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